from heating, the changes of temperature during the outflow process are negligible. The contact surface near the open end shows that during the suction process the external medium reaches the depth ~ 0.38 of the length of the cylinder. The distribution of pressure was used to determine the dependence of pressure on time, which is shown in Fig. 2.

NOTATION

x is the coordinate along the axis of the cylinder; t, time; u, velocity; p, pressure; ρ, density; k, exponent of the adiabatic curve; χ, wetted perimeter of the cylinder; F, cross-sectional area of the cylinder; τ_0, force of friction of the liquid against the cylinder walls per unit area; λ, friction coefficient; r_h, hydraulic radius of the cylinder; μ, coefficient of dynamical viscosity; Re, Reynolds number; and a, speed of sound. The subscript 0 refers to the conditions of the external medium, and 1 to the perturbed conditions.

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HEAT TRANSFER BETWEEN THREE MEDIA IN TRIPLE COUNTERCURRENT PIPE FLOW

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A method is discussed for calculating the thermodynamic characteristics associated with the interaction of three fluid flows in pipes. Analytical relations are derived for the case of triple concentric countercurrent flow.

The calculation of heat transfer between two media flowing in the same or opposite directions does not present any difficulties. Recently, however, there has been a growing number of problems in which it is required to calculate heat transfer in the simultaneous interaction of three flows, but the literature does not offer analytical solutions for determining the temperature profile along the flow axis, the quantity of heat transmitted across the separating surface, and other characteristics. The number of combinations along the relative direction of motion of the media and in the direction of heat transfer can be enormous in this case. The most complicated situation in this class of problems is triple concentric countercurrent flow of the media.

In particular, e.g., the recent efforts aimed at intensifying petroleum recovery have created the important problem of supplying heat to oil-bearing strata at great depths. One of the more promising methods of solving this problem is to create a deep underground steam generator of adequate output, situated in the downhole zone, with injection of the generated steam into the oil stratum. The specific attributes of this problem are that, first, it is required to lower the steam generator through a drivepipe with a diameter of 150-200 mm and, second, to supply the steam generator with air, fuel, and water and to exhaust the combustion products to the surface of the earth. At the same time, it is necessary to maintain the

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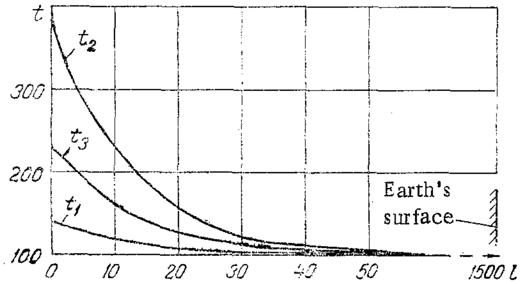

Fig. 1. Axial temperature distribution in the concentric countercurrent thermal interaction of three flows (water and air input temperatures at l = 1500 m; $t_1 = t_3 = 100°C$; input temperature of combustion products at l = 0; $t_2 = 400°C$).

outer wall of the pipe system at a minimum temperature so as not to heat up the underground column.

We have developed a procedure for calculating the heat transfer associated with the annular concentric countercurrent flow of three media. The problem is investigated in the specific example of the system of pipes for a downhold steam generator, but the approach to the solution of the problem and the resulting conclusions and relations, of course, are of a general nature.

Let there be given a system of three coaxial pipes. Water and air flow, respectively, in the inner pipe and the annular duct formed by the middle and outer pipes. Flowing in the opposite direction along the annular duct formed by the inner and middle pipes are the combustion products. The heat of the combustion products is transmitted to the water and air. We neglect the thermal resistances of the walls separating the flows, and the heat transfer from the outer pipe into the surrounding medium. Radial temperature gradients are not present in the ducts.

Basic Heat-Transfer Equations. The quantity of heat acquired by the water in a section of pipe of length dl from the combustion products (see Fig. 1) is

$$dQ_1 = K_1 \pi D_1 (t_2 - t_1)\, dl. \tag{1}$$

The quantity of heat acquired by the air is

$$dQ_2 = K_2 \pi D_2 (t_2 - t_3)\, dl. \tag{2}$$

The quantity of heat removed in heat transfer from the combustion products to the air and water in the section dl is

$$dQ = -(dQ_1 + dQ_2). \tag{3}$$

On the other hand (for motion along the l axis; see Fig. 1),

$$dQ = c_2 G_2 dt_2, \tag{4}$$

$$dQ_1 = -c_1 G_1 dt_1, \tag{5}$$

$$dQ_2 = -c_3 G_3 dt_3. \tag{6}$$

Method of Solution. From Eqs. (1)-(6) it follows that

$$dt_1 = -\frac{K_1 \pi D_1}{c_1 G_1} (t_2 - t_1)\, dl, \tag{7}$$

$$dt_2 = -\frac{K_1 \pi D_1}{c_2 G_2} (t_2 - t_1)\, dl - \frac{K_2 \pi D_2}{c_2 G_2} (t_2 - t_3)\, dl, \tag{8}$$

$$dt_3 = -\frac{K_2 \pi D_2}{c_3 G_3} (t_2 - t_3)\, dl. \tag{9}$$

We introduce the variables $y = t_2 - t_1$ and $x = t_2 - t_3$ and deduce the following relations from Eqs. (7)-(9):

$$\frac{dy}{dl} = -Bx - (A - D)y, \tag{10}$$

$$\frac{dx}{dl} = -(B - C)x - Ay. \tag{11}$$

Here

$$A = \frac{K_1 \pi D_1}{c_2 G_2}; \quad B = \frac{K_2 \pi D_2}{c_2 G_2}; \quad D = \frac{K_1 \pi D_1}{c_1 G_1}; \quad C = \frac{K_2 \pi D_2}{c_3 G_3}.$$

We seek a solution of the system of differential equations (10), (11) in the form

$$y = A_1 \exp(\alpha_1 l) + B_1 \exp(\beta_1 l), \tag{12}$$

$$x = A_2 \exp(\alpha_2 l) + B_2 \exp(\beta_2 l). \tag{13}$$

It must satisfy the differential equations themselves as well as the boundary conditions.

On the basis of expressions (12) and (13), Eqs. (10) and (11) acquire the form

$$A_1 \alpha_1 \exp(\alpha_1 l) + B_1 \beta_1 \exp(\beta_1 l) = -A_1 (A - D) \exp(\alpha_1 l) - \\ - B_1 (A - D) \exp(\beta_1 l) - A_2 B \exp(\alpha_2 l) - B B_2 \exp(\beta_2 l), \tag{14}$$

$$A_2 \alpha_2 \exp(\alpha_2 l) + B_2 \beta_2 \exp(\beta_2 l) = -A A_1 \exp(\alpha_1 l) - \\ - A B_1 \exp(\beta_1 l) - (B - C) A_2 \exp(\alpha_2 l) - B_2 (B - C) \exp(\beta_2 l). \tag{15}$$

These relations represent identities, which are valid for any values of l. They can be satisfied only under the condition that

$$\alpha_1 = \alpha_2 = \alpha, \quad \beta_1 = \beta_2 = \beta, \tag{16}$$

$$A_1 (\alpha + A - D) + A_2 B = 0, \tag{17}$$

$$B_1 (\beta + A - D) + B B_2 = 0, \tag{18}$$

$$A A_1 + A_2 (\alpha + B - C) = 0, \tag{19}$$

$$A B_1 + B_2 (\beta + B - C) = 0. \tag{20}$$

From Eqs. (17)-(20) we deduce two identity relations, which yield quadratic equations in α and β:

$$\frac{\alpha + A - D}{A} = \frac{B}{\alpha + B - C}, \tag{21}$$

$$\frac{\beta + A - D}{A} = \frac{B}{\beta + B - C}. \tag{22}$$

The roots of these equations are

$$\alpha(\beta) = \frac{-b \pm \sqrt{b^2 - 4d}}{2}, \tag{23}$$

whence we infer that

$$\alpha = \frac{-b + \sqrt{b^2 - 4d}}{2}, \tag{24}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4d}}{2}; \tag{25}$$

here $b = A - D + B - C$ and $d = DC - AC - DB$.

The foregoing arguments are valid under the condition $b^2 > 4d$.

From (17), (19) and, respectively, (18), (20) we have

$$A_2 = A_1 \frac{D - \alpha}{C - \alpha}, \tag{26}$$

$$B_2 = B_1 \frac{D-\beta}{C-\beta}. \quad (27)$$

We now integrate Eqs. (7)-(9) to obtain

$$t_1 = -\frac{DA_1}{\alpha} \exp(\alpha l) - \frac{DB_1}{\beta} \exp(\beta l) + \text{const}, \quad (28)$$

$$t_2 = -\frac{A_1(D-\alpha)}{\alpha} \exp(\alpha l) - \frac{B_1(D-\beta)}{\beta} \exp(\beta l) + \text{const}, \quad (29)$$

$$t_3 = -\frac{CA_1}{\alpha}\frac{D-\alpha}{C-\alpha} \exp(\alpha l) - \frac{CB_1}{\beta}\frac{D-\beta}{C-\beta} \exp(\beta l) + \text{const}. \quad (30)$$

To evaluate the constants of integration, we use the boundary conditions (l_K is the total length of the system)

$$l = l_K: \; t_1 = t_{1,K} = h, \; t_3 = t_{3,K} = f;$$
$$l = 0: \; t_2 = t_{2,0} = g.$$

Then the equations for the temperatures of the flows along the length of the pipe system acquire the form

$$t_1 = h + \frac{DA_1}{\alpha} [\exp(\alpha l_K) - \exp(\alpha l)] + \frac{DB_1}{\beta} [\exp(\beta l_K) - \exp(\beta l)], \quad (31)$$

$$t_2 = g + \frac{A_1(D-\alpha)}{\alpha} [1 - \exp(\alpha l)] + \frac{B_1(D-\beta)}{\beta} [1 - \exp(\beta l)], \quad (32)$$

$$t_3 = f + \frac{CA_1}{\alpha}\frac{D-\alpha}{C-\alpha} [\exp(\alpha l_K) - \exp(\alpha l)] + \frac{CB_1}{\beta}\frac{D-\beta}{C-\beta} [\exp(\beta l_K) - \exp(\beta l)]. \quad (33)$$

The next problem is to determine the coefficients A_1 and B_1. Comparing the quantities $y = t_2 - t_1$ and $x = t_2 - t_3$ and making use of Eqs. (31)-(33), (12), (13), and (26), (27), we obtain the system of equations

$$FX_1 + GY_1 = h - g, \quad (34)$$
$$FX_2 + GY_2 = f - g, \quad (35)$$

in which

$$F = \frac{A_1}{\alpha} \exp(\alpha l_K), \quad G = \frac{B_1}{\beta};$$

$$X_1 = (D-\alpha)\exp(-\alpha l_K) - D;$$

$$Y_1 = D - \beta - D\exp(\beta l_K);$$

$$X_2 = (D-\alpha)\exp(-\alpha l_K) - C\frac{D-\alpha}{C-\alpha};$$

$$Y_2 = D - \beta - C\frac{D-\beta}{C-\beta} \exp(\beta l_K).$$

Solving the system (34), (35) for F and G, we find the coefficients A_1 and B_1.

The equations for the temperature distribution along the length for triple countercurrent flow can be written in the final form

$$t_1 = h + DF\{1 - \exp[\alpha(l - l_K)]\} + DG[\exp(\beta l_K) - \exp(\beta l)], \quad (36)$$

TABLE 1. Longitudinal Flow-Temperature Distribution (°C) for Various Boundary Conditions

l, m	0	20	50	100	500	1000	1400	1450	1480	1500
					$h=100,\ g=400,\ f=50$					
t_1	135	103	96	95	95	95	95	95	97	100
t_2	400	155	97	95	95	95	95	94	89	81
t_3	233	122	96	95	95	95	95	91	77	50
					$h=100,\ g=600,\ f=50$					
t_1	161	108	96	95	95	95	95	95	97	100
t_2	600	195	104	95	95	95	95	95	89	81
t_3	323	140	99	95	95	95	95	91	77	50
					$h=100,\ g=400,\ f=100$					
t_1	139	108	101	100	100	100	100	100	100	100
t_2	400	159	105	100	100	100	100	100	100	100
t_3	235	127	102	100	100	100	100	100	100	100
					$h=100,\ g=600,\ f=150$					
t_1	170	118	106	105	105	105	105	105	103	100
t_2	600	203	114	105	105	105	105	106	111	119
t_3	328	149	109	105	105	105	105	109	123	150
					$h=150,\ g=500,\ f=100$					
t_1	187	152	146	146	146	146	146	146	147	150
t_2	500	202	149	146	146	146	145	144	140	131
t_3	305	171	147	146	146	146	145	142	129	100

$$t_2 = g + (D - \alpha) F \{\exp(-\alpha l_к) - \exp[-\alpha(l_к - l)]\} + G(D - \beta)[1 - \exp(\beta l)], \quad (37)$$

$$t_3 = f + CF\frac{D-\alpha}{C-\alpha}\{1 - \exp[-\alpha(l_к - l)]\} + CG\frac{D-\beta}{C-\beta}[\exp(\beta l_к) - \exp(\beta l)]. \quad (38)$$

The quantity of heat transmitted to the water is determined by means of Eq. (1):

$$Q_1 = K_1 \pi D_1 \int_0^{l_к} (t_2 - t_1)\, dl. \quad (39)$$

The heat acquired by the air is determined from Eq. (2):

$$Q_2 = K_2 \pi D_2 \int_0^{l_к} (t_2 - t_3)\, dl. \quad (40)$$

As an illustration, Table 1 gives the results of calculations of the temperature profile according to the relations derived above for several different sets of boundary conditions and certain dimensional, mass-flow, and heat-transfer characteristics: D_1 = 0.06 m; D_2 = 0.10 m; G_1 = 4000 kg/h; G_2 = 3000 kg/h; G_3 = 2800 kg/h; c_1 = 1.00 kcal/kg°C; c_2 = 0.28 kcal/kg°C; c_3 = 0.25 kcal/kg°C; K_1 = 260 kcal/m²h°C; K_2 = 150 kcal/m²h°C; l_K = 1500 m.

Several conclusions can be drawn from an inspection of the results under the given conditions. In particular:

1) The flow temperatures equalize over fairly short intervals at the ends of the pipes (∼50 m).

2) The temperature of the air at the entry to the system only comparatively slightly affects the temperature level over the entire length of the pipes, while the influence of the water temperature is more significant.

3) The influence of the initial temperature of the combustion products is felt only in the entry section (∼50 mm).

The proposed method can be used to derive analytical relations for calculating the temperature fields and other characteristics for other possible combinations of relative motion of the media and the direction of heat transfer between flows.

NOTATION

D_1 is the median diameter of inner pipe; D_2, median diameter of middle pipe; t_1, temperature of water; t_2, temperature of combustion products; t_3, temperature of air; G_1, mass flow of water; G_2, mass flow of combustion products; G_3, mass flow of air; c_1, specific heat of water; c_2, specific heat of combustion products; c_3, specific heat of air; k_1, coefficient of heat transfer from combustion products to water; K_2, coefficient of heat transfer from combustion products to air; and l_K, length of pipe system.